IRRATIONAL THINKING – A SACRED GEOMETRY PRIMER

by Grahame Gardner

It has been previously shown in these pages how nature seems to favour the use of what are termed irrational numbers in her work. As I explained in a previous article[1], these are numbers like Pi (Π) and Phi (φ), where the decimal part is infinitely long and non-repeating. Pi is also a special kind of irrational number called a transcendental number, but we won't go into that here because it's defined more by what it isn't than what it is, and in any case it's not relevant to this discussion[2].

Some irrational numbers are so commonly known that they have names; Pi, Phi, Feigenbaum's constant and the Comma of Pythagoras are just a few examples. Others are known only by their function; the square roots of 2, 3, and 5 for instance. They are found to be an inherent component of many natural forms and structures, and form an integral part in the design of sacred spaces. From Stonehenge to Chartres cathedral, from the Parthenon to the Great Pyramid, you will find irrational numbers underpinning the layout of the space. I say underpinning, because they are never openly manifest. They appear only in the relationship of one quantifiably measurable part to another (another name for irrationals is incommensurate numbers). To understand the importance of these numbers, we need to know something about the noble and ancient art of Sacred Geometry.

Sacred Geometry deals with our perception and definition of space. It is the Universal framework whereby the spiritual manifests into the material. Spaces constructed using the principles of sacred geometry act as a bridge between the worlds, and sacred geometric forms naturally produce dowsable energy fields. It makes no difference if it's a chalk circle on your living-room carpet or the dizzying architecture of a Gothic cathedral; the principles are exactly the same. To the masons who raised the great Gothic cathedrals and other places of worship, sacred geometry was of paramount importance to the construction; indeed no religious establishment could be expected to function properly without it. The spaces are designed to be uplifting to the spirit, to resonate on a subconscious level in such a way that the possibility of a spiritual connection is maximised. An impressive side effect of sacred geometry is that many of these structures possess extraordinary acoustic properties, a result of the inter-related harmonic structure behind both music and geometry. Certain types of music (e.g. Bach) were designed to be sacred geometry you can hear, so it is little wonder the buildings resonate with it.

In ancient times it was believed that numbers are the underlying reality behind all things. All things were linked through number and could be manifested through number. Number expressed in time is music, number expressed in space is geometry, and number expressed in space-time is astrology. Just as astrology can provide us with a map of consciousness, so can geometry. These concepts underlie the entire world-view of the ancients but have largely been forgotten in our modern hyper-rational scientific orthodoxy. We've lost sight of the spiritual qualities of number and shape by our emphasis on brute quantity and hard computation. We're taught to see numbers as mere quantities, instead of possessing qualities and characters with distinct personalities, resonating with each other and Universe in a harmonious cosmic dance (it used to be called Music of the Spheres). And this is why irrational numbers are so fascinating to the geometer; they cannot ever be expressed as a quantity, they can only be appreciated as qualities. They are the inner, esoteric face of number, becoming visible only in the relationships between parts. I think of them as the cracks between the paving stones of reality. The Pythagoreans of ancient Greece, who are credited with discovering the infinite nature of these numbers in the sixth century B.C., were so puzzled and awe-struck by their discovery that they tried to keep it a secret by proscribing the death penalty for those who would divulge it[3].

We have seen that Sacred Geometry is interested in *irrational* numbers. These numbers are the cornerstones of sacred geometry because they symbolically manifest the infinite in normal space. For a practical example of this, let's take a look at an ancient example of sacred space, the King's

Chamber in the Great Pyramid. The shape of this is a form known as a *double cube*. That is, the long sides are twice the length of the short sides. Consider just the floor area for the moment; this would be a *double-square* rectangle (Fig. 1). The important thing about the double-square is the diagonal. If we say that the short side of the figure has a length of one unit, then the long side has a length of 2 units (remember in sacred geometry we're only interested in proportions and ratios, not actual measurements). That means that a diagonal (d) of this rectangle will have a length equal to the square root of five (by Pythagoras' theorem: $d^2 = 2^2 + 1^2 = 4 + 1 = 5$, therefore $d = \sqrt{5}$).
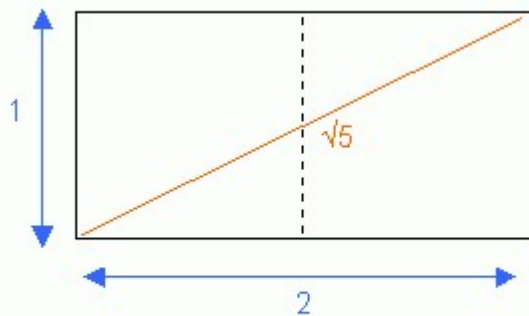

Fig. 1. Double-square rectangle

Figure 1

Now the square root of five (2.236…) is another one of those irrational numbers that can't be calculated precisely. It goes on forever, never repeating, always changing. So it can be quite accurately said that in a sense, you cannot ever measure this diagonal exactly. It represents the infinite. Both diagonals of this rectangle are root 5, so if you were to stand exactly at the centre of this double-square rectangle, you stand in the centre of a harmoniously proportioned space, but you also stand at the crossing point of two diagonals of theoretically infinite length. What better space to commune with the One?

Let's take a look at some other basic shapes and ratios as we take a tour of the numbers One through Five. Along with Pi (Π) and Phi (φ), the main irrationals used are the square roots of 2, 3 and 5, and combinations thereof, like the $\sqrt{2}:\sqrt{3}$ proportions prevalent in Chartres cathedral[4]. It's too large a subject to cover fully here, but this will give you a basic grasp of the important concepts involved.

The Circle:

Simplest shape of all, and manifestation of the One. Draw a circle around yourself and you immediately distinguish your personal space from your surroundings. That's really the power of the circle. It is the first expression of Universe; a horizon, a boundary between Self and Other. Defined as 'an infinite number of points equidistant from a centre', the circle in its manifestation implies the divine generation of shape and form from nothing to everything. The irrational number that the circle generates is, of course, Pi (=3.1415926…). The circumference is calculated from the radius by the formula 2Πr, where r is the radius of the circle. Because of the use of Pi, we cannot ever know the value of both the radius and the circumference in whole-number units. If either the radius or circumference is measurable in whole, rational units, then the other will always be an endless, irrational decimal. Thus the circle represents the limited and the limitless in one body.

Examples of sacred spaces based upon the circle are Stonehenge, Ring of Brodgar in Orkney, and the Merry Maidens circle in Cornwall. Most of what we call 'stone circles' are not true circles, but more complex geometrical forms. A more accurate term would be 'stone rings', and you do see this being used in some literature today. Some researchers think that the complicated compound outlines of some rings were attempts to produce whole-number circumferences and radii in a quest for sacred geometric perfection; however research by the EEG seems to indicate that the ring will have greater beneficial earth energy effects the closer the perimeter divided by the sum of the two axes approximates the Golden Mean of 1.61803…[5].

The Vesica Piscis:

The circle divides, the One becomes Two. Something generates Other. The circle replicates by contemplating itself, reflecting its light, and casting an identical shadow. Now we have two separate qualities, but they are still in the Void, distinct and without interaction. Nothing can happen until they merge, and then we have a concept of Three-ness and true manifestation can begin. To the ancients, Three was the first true number; One and Two were seen as abstract principles, unmanifest. The shape formed by two overlapping circles is called the vesica piscis (Fig. 2). The vesica piscis embodies all the concepts of duality that you can think of; the two circles both attract and repel each other, giving polarity and tension, whilst the overlapping area gives us a portal of manifestation.
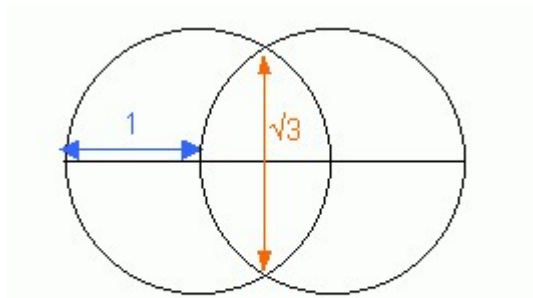

Fig. 2. Vesica Piscis

Any two circles that overlap will produce a vesica (the overlapping area), but only two circles of identical size whose centres are located on the circumference of the other produce a vesica piscis (it's Latin for 'bladder of the fish'). This ancient symbol is significant in esoteric Christian lore, and forms the basis for the 'fish' sigil that was used initially as a secret sign between early Christians and is now seen most often decorating car bumpers. But the symbol is much older than Christianity. The vesica doesn't just represent a fish bladder; it's really about the birth portal, the cosmic Yoni of the Goddess. Imagine looking up from between her outstretched thighs and you'll get the picture. All subsequent geometric shapes can be produced through the portal of the vesica using the geometer's tools, but regrettably that is beyond the scope of this article.

The irrational number produced by the vesica piscis is the square root of 3. If the radius of each of the circles is 1 unit, then a vertical line drawn down the centre of the vesica has a length of root 3 ($\sqrt{3}=1.7320…$).

The vesica piscis crops up a lot in sacred architecture, but perhaps its most obvious manifestation is the Gothic arch, bastion of so many of our spiritual buildings. A Gothic arch is basically the vesica part with vertical extensions from the centre of the sides. Less obvious manifestations of the vesica are in the floor plans of many churches and chapels where the vesica defines the dimensions of the rectangular layout. The Mary chapel in Glastonbury Abbey is said to be the most perfect example of this[6]. The vesica can also be seen as representative of the shape of the human aura, and is used as such in many religious paintings. This has an interesting symbolism; if the two circles of the vesica piscis are seen as Heaven and Earth, then the vesica part represents the bridge between the two – or in other words, Jesus and the established Church. This is why you often see the figure of Christ enclosed in a vesica-shaped aura on the front of churches[7]).

The Square:

The Square symbolises the imposition of structure upon the earth. Instead of the limitless circle of the One, we now have the orientation and implied directions of the Four. Four quarters. Four directions. Four winds. Four elements… and so on. We're perhaps more familiar with this shape than any other, since it permeates our lives in practically everything we build or make (Fig 3).
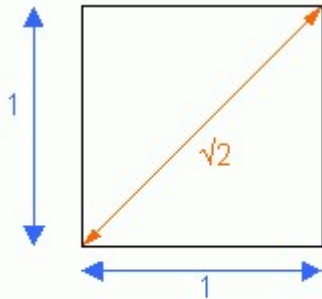
Fig. 3. The Square

In working with sacred space, the important bit of the square is the diagonal. If the square has sides of 1 unit in length, then the length of the diagonal is the square root of 2 ($\sqrt{2}$=1.41421…). Again, this is an irrational number. Stand in the centre of the square and you stand at the centre of two theoretically infinite lines. A classical example this form was the Holy of Holies in Solomon's Temple. That's the bit that nobody was allowed into, where they kept the Ark. A more contemporary incarnation is the Kaaba in Mecca.

The combination of the square and the circle represents the fusion of heaven and earth, and 'squaring the circle' is regarded as the pinnacle of the sacred geometer's art. This means producing a circle overlaying a square such that either the circumference of the circle equals the perimeter of the square, or the area of the circle equals that of the square. Many early religious buildings were designed using this squared-circle geometry, perhaps most famously the Hagia Sophia in Istanbul, one of the oldest Christian churches in existence, commissioned by the Emperor Justinian and completed in 532 c.e. Its square form with staggering 180-foot high hemispherical dome is still a wonder to behold. It was the largest enclosed space on the planet for over a thousand years. Interestingly, when Constantinople fell to the Turks, the building was converted into a mosque and the beautiful gold mosaics of the interior painted over. But its form was considered so perfect that all subsequent mosques in Istanbul were modelled on it, and so the squared circle form was introduced to Islam. Many years later, the West took the design of the Gothic arch from Islamic architecture, so it seems a fair trade!

The Double Square:

As it sounds, two squares side by side. A rectangle with short side of 1 unit and long sides of 2. This represents the 2:1 ratio of the octave in musical terms. We looked at this shape earlier as the basis of the King's Chamber in the Great Pyramid (Fig. 1); it's also the shape of 'The Holy Place' in Solomon's Temple. That was the main part of the temple, not quite as sacred as the Holy of Holies.

The Pentagon/Pentagram:

The four terms of the square are enough to account for the idea of matter, or substance. But when we get to Five we introduce the concept of Spirit, or governing intelligence; in other words Life. Five is seen as the union of Two (female) and Three (male). Thus the five-sided pentagon and in particular the pentagram star within it symbolises life and regeneration. Because if its self-symmetry and use of the Golden Proportion[8], the pentagram contains within itself the seed of it's own replication in progressively smaller or larger scales (Fig. 4). Today we call this fractal geometry, and this recursive template is found throughout nature.
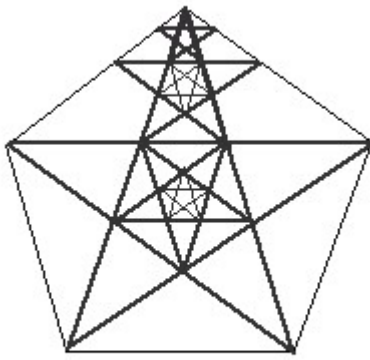
Fig 4: Fractal

Archaeologists have found pentagrams on Mesopotamian potsherds dating back to 3500 BCE. Pentagrams also appear in ancient Egyptian, Greek and Roman art. The use of the pentagram in Christian sacred geometry is due in no small part to the writings of Hildegard of Bingen, the twelfth century Benedictine nun and abbess. For her, the pentagram was the central symbol of the microcosm, the reflection on Earth of the divine plan and the divine image. Hildegard saw the pentagram as representing the human form because we have five senses – sight, smell, hearing, taste, and touch; and five extremities – two legs, two arms and a head. And, because humankind was made in God's image, she also saw the pentagram as representing God.

Other Christians saw the symbol as representing the five wounds of Christ and, as such, it was considered a potent protection against evil. Earlier Hebrew tradition associated the pentagram with the Pentateuch, the first five books of the bible. In the late Middle Ages, the pentagram became a symbol of knightly virtues. In the poem Sir Gawain and the Green Knight, the points of the pentagram symbolise chastity, chivalry, courtesy, generosity, and piety.

During the Inquisition, the inverted pentagram became associated with evil and the Devil. The pentagram was thought to represent the head of a goat, the devil, or a witch's foot. Regrettably the pentagram, whether upright or inverted, still carries this negative association today for many people, but it is a gross debasement of such a truly Divine symbol.

The most obvious example of space constructed on pentagonal principles is, of course – the Pentagon in Washington DC. Here the form is utilised more for its defensive and protective aspects. Another example is the Universal Hall of the Findhorn Foundation. But pentagonal geometry can be found lurking just beneath the surface of many sacred spaces, perhaps defining the layout of a church apse or the pattern of a rose window.

So the next time you visit a Gothic cathedral, pause a while and reflect upon the supreme skills of the Master who designed and laid out the entire space using little more than the straightedge, compasses and druid's cord of the sacred geometer.

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1.   *'The Golden Proportion'*, EEG newsletter March 2002 []

2.   An irrational number cannot be defined as a fraction p/q for any integers p and q, i.e. it cannot be written as one whole number divided by another. A number is called algebraic if it is the root of a polynomial (of any degree) with rational coefficients. Any number that is non-algebraic is called transcendental. []

3.   György Doczi: 'The Power of Limits' p5 []

4.   John James: 'The Master Masons of Chartres' p39 []

5.   Jim Lyons: 'The Ubiquitous Ovoid' BSD EEG Newsletter, March 2002 []

6.   Gordon Strachan: 'Jesus the Master Builder' p201 []

7.   Robert Ferre: 'Sacred Geometry' (lecture tape []

8.   'The Golden Proportion', EEG newsletter March 2002 []

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